Study Guide

Multiplication and Division of Radicals
02/29/2012

Radicals: Multiplication/Division
A radical sign looks like a check mark with a line across the top. The radical sign is used to communicate square roots. The skill Multiplying/Dividing Radicals evaluates the ability to find the product or quotient of 2 or more radicals.

To multiply radicals, multiply the numbers in front of the radical sign and the numbers which make up the radicand (the number or numbers under the radical sign).

Example 1:

(1) $\left(6\sqrt{5}\right)\left(9y\sqrt{6}\right)$

(2) $\left(6 \cdot 9y\right)\left(\sqrt{5} \cdot \sqrt{6}\right)$
$54y\left(\sqrt{5} \cdot \sqrt{6}\right)$

(3) $54y\left(\sqrt{5 \cdot 6}\right)$
$54y\sqrt{30}$

Step 1: Identify the expressions to be multiplied.
Step 2: Multiply the numbers in front of the radical sign.
Step 3: Multiply the radicands, leaving the product as a radicand.

The answer is $54y\sqrt{30}.$

When dividing radicals, the problem can be set up as a fraction. If the denominator of this fraction contains a radical, the denominator must be rationalized. To do this, multiply the numerator and denominator by the radical found in the denominator. As a result, the numerator may now contain a radical, but the denominator will not. Simplify the numerator, and then simplify the entire fraction if possible, and the process is complete.

Example 2:

(1) $\frac{7\sqrt{32}}{5\sqrt{63}} = \frac{7\sqrt{32}}{5\sqrt{63}} \cdot \frac{\sqrt{63}}{\sqrt{63}} = \frac{7\sqrt{32 \cdot 63}}{5 \cdot 63}$

(2) $\frac{\not{7}\sqrt{32 \cdot 63}}{5 \cdot \not{63}} = \frac{\sqrt{32 \cdot 63}}{45}$

(3) $\frac{\sqrt{4^2 \cdot 2 \cdot 3^2 \cdot 7}}{45} = \frac{4 \cdot 3\sqrt{2 \cdot 7}}{45} = \frac{12\sqrt{14}}{45}$

(4) $\frac{4\sqrt{14}}{15}$

Step 1: Multiply the numerator and denominator by the radical to leave a whole number in the denominator.
Step 2: Simplify by dividing the 7 into 63.
Step 3: Look for perfect squares factors. $32 = 4^2 \cdot 2$ and $63 = 3^2 \cdot 7$
Step 4: Simplify. A radical is simplified when it contains no perfect squares.

Example 3: Multiply the expressions.

$(2\sqrt{5} + 4\sqrt{11})(3\sqrt{7} - 5\sqrt{3})$

(1) $(2\sqrt{5}\cdot 3\sqrt{7})+(2\sqrt{5}\cdot -5\sqrt{3})+(4\sqrt{11}\cdot 3\sqrt{7})+(4\sqrt{11}\cdot -5\sqrt{3})$

(2) $6\sqrt{35}-10\sqrt{15}+12\sqrt{77}-20\sqrt{33}$

Step 1: Multiply the first two terms, the outer two terms, the inner two terms, and the last two terms.
Step 2: Multiply the numbers outside the radicals, then multiply the numbers inside the radicals of each term. Simplify, if possible.

When rationalizing the denominator of a fraction, multiply the numerator and denominator by the conjugate of the denominator.

Example 4: Rationalize.

$\frac{5}{\sqrt{5}-2}$

(1) $\frac{5}{\sqrt{5}-2}\bullet\frac{\sqrt{5}+2}{\sqrt{5}+2}$

(2) $\frac{5\bullet\sqrt{5}+5\bullet 2}{\sqrt{5}\bullet\sqrt{5}+2\bullet\sqrt{5}-2\bullet\sqrt{5}+2\bullet -2}$

(3) $\frac{5\sqrt{5}+10}{5+2\sqrt{5}-2\sqrt{5}-4}$

$\frac{5\sqrt{5}+10}{5-4}$

(4) $\frac{5\sqrt{5}+10}{1}$

(5) $5\sqrt{5}+10$

Step 1: Multiply the numerator and denominator by the conjugate of the denominator.
Step 2: Begin to simplify the numerator by multiplying radical 5 and 2 by 5. Begin to simplify the denominator by multiplying the binomials.
Step 3: Complete all multiplications, then add all like terms.
Step 4: Complete the simplification of the denominator by subtracting 4 from 5.
Step 5: Divide both terms in the numerator by the number in the denominator.

Answer: $5\sqrt{5}+10$